In-medium relativistic kinetic theory and nucleon-meson systems

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Within the $\sigma - \omega$ model of coupled nucleon-meson systems, a generalized relativistic Lenard–Balescu–equation is presented resulting from a relativistic random phase approximation (RRPA). This provides a systematic derivation of relativistic transport equations in the frame of nonequilibrium Green's function technique including medium effects as well as fluctuation effects. It contains all possible processes due to one meson exchange and special attention is kept to the off–shell character of the particles. As a new feature of many particle effects, processes are possible which can be interpreted as particle creation and annihilation due to in-medium one meson exchange. In-medium cross sections are obtained from the generalized derivation of collision integrals, which possess complete crossing symmetries.

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I. INTRODUCTION

Since Walecka [1] has established his model of meson exchange already 20 years ago, field theoretical models were successfully applied for the description of heavy-ion collisions at intermediate energies [2–5]. Thereby the derivation of transport equations starting from a microscopic model is paid great attention [6–8].

The most powerful and appropriate method to describe quantum many particle systems under extreme conditions like nuclear matter or condensed stellar objects are the real-time-Green functions technique. Many excellent reviews are published concerning this method during the past decade [6,7,9–11]. Moreover there are many investigations in classical relativistic treatment [12,13]. Several papers are devoted the formulation of transport equations in quantum field theory [6,8,14,15].

In this paper we like to present briefly a derivation of relativistic transport equations for the $\sigma - \omega$ model. The formal development of relativistic transport models is well known and documented in the literature [16,17]. Here special attention is kept to the description of fluctuations and the influence of medium effects on the kinetic equation. Additional reaction channels are opened by the influence of many particle effects, which would be forbidden in uncorrelated systems. Further, we derive a decoupling between nucleon and meson-equations, which result in an effective squared one-meson exchange potential containing no mixed terms of meson contributions. This is established by the use of generalized optical theorems.

The outline of the paper is the following. In chapter 2 we review the fundamental equations of the Yukawa Lagrangian and introduce the nonequilibrium Green's function technique based on the very fundamental principle of weakening of initial correlation. The structure of equations are developed in the Schwinger formalism, generalized to the set of four nonequilibrium Green's functions as it was similar done by Bezzerides [18] and recently by Davis [17]. The spectral information and therefore the quasiparticle properties are discussed in chapter 3 which establish a quite natural generalization of Bruckner theory by the aspect of complete particle – antiparticle symmetry of interacting quasiparticles [6].

To set up a consistent kinetic theory an equation for the Wigner distribution function should be available, which includes all features of relativistic quasiparticle behaviour like Pauli-blocking, screening and relativistic effects, such as scattering between particles and antiparticles and pair creation and destruction processes. The consequent treatment of many particle effects adopted in section 4 as it was done in the nonrelativistic case [19]. The generalization of collision integrals include the effect of density fluctuations by an dynamical meson exchange potential and describes completely the particle-antiparticle symmetry. Particularly, it contains Pauli blocking and inelastic processes by the medium. A dynamical potential enters the equation due to density fluctuations and the equation can be considered therefore as a generalization of quantum mechanical Boltzmann equation [20] to a Lenard-Balescu-type one. This is the main result of the present paper and should be the starting point to kinetic description of hot nuclear matter

instead of the relativistic Vlassow treatment [21]. Similar equations have been obtained in T – Matrix approximation but without pair creation and dynamically screening by De Boer [22], De Groot [13], Botermans [6] and Malfliet [23].

From the found collision integrals we derive in-medium cross–sections, which differ from ordinary Born approximation by two facts. Firstly, no mixed coupling terms occur between different kinds of meson contributions. This is due to the more general approximation of self energies instead of the decoupling normally used. Secondly, the medium effects the cross section by an energy shift from the vector-meson exchange and the renormalized effective mass by the scalar-meson contribution.

II. MODEL AND BASIC EQUATIONS

The Yukawa Lagrangian, which couples scalar and vector meson fields to fermion fields describes the simplest form of a model for nuclear matter [1]

$$
\begin{aligned}
L = \quad & -\overline{\Psi}\left(-i\gamma^{\mu}\partial_{\mu} + \kappa_{0}\right)\Psi + \\
& + \frac{1}{2}\partial_{\mu}\Phi\partial^{\mu}\Phi - \frac{1}{2}m_{s_{o}}^{2}\Phi^{2} + :\, g_{s}\overline{\Psi}\Phi\Psi\,: \\
& - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_{v_{o}}^{2}\varphi_{\lambda}\varphi^{\lambda} + :\, g_{v}\overline{\Psi}\gamma_{\lambda}\Psi\varphi^{\lambda}\,:\,.
\end{aligned} \tag{1}
$$

Here m_{s_0}, m_{v_0} are the bare scalar and vector meson mass and κ_0 is the bare nucleon one respectively. Further, :: denotes the normal ordering procedure to ensure the renormalization of baryon density [24]. After renormalization of the masses the coupling constants have to be chosen in such a way, that on one hand, the equilibrium ground state properties can be fitted, and on the other hand, the scattering data can be reproduced [4,25]. Because of the renormalizability of the model Lagrangian we can choose for the occurring divergent vacuum terms a standard procedure resulting in physical masses. For our used selfconsistent Hartree-Fock and RPA approximation this is well known in literature [24]. In the following we use therefore already the renormalized fields and masses, which may be considered as an effective model.

From (1) the equations of motion read

$$
\left(\square + m_{s}^{2}\right)\Phi = g_{s}\,:\,\overline{\Psi}\Psi\,:\, \equiv j_{\Psi} \tag{2}
$$

$$
\left(\square + m_{v}^{2}\right)\varphi_{\nu} = g_{v}\,:\,\overline{\Psi}\gamma_{\nu}\Psi\,:\, \equiv j_{\Psi v} \tag{3}
$$

$$
\left(i\gamma^{\mu}\partial_{\mu} - \kappa\right)\Psi = -g_{s}\,:\,\Psi\Phi\,:\, +\, g_{v}\,:\,\gamma^{\lambda}\varphi_{\lambda}\Psi\,:\, \equiv j_{\varphi}. \tag{4}
$$

With the help of the Green's function G_{o}^{s}, G_{0}^{v} for the free meson equation (2, 3) the mesonic degree of freedom in eq. (4) can be eliminated [6,26,27]

$$
\left(i\gamma^{\mu}\partial_{\mu} - \kappa\right)\Psi_{\alpha} = \left[g_{v}^{2}\left(\gamma_{\mu}\right)_{\alpha\beta}\left(G_{0}^{v}\right)^{\mu\rho}\left(\gamma_{\mu}\right)_{\delta\theta} - g_{s}^{2}G_{o}^{s}\right]\bar{\Psi}_{\delta}\Psi_{\theta}\Psi_{\beta} \tag{5}
$$

Here a greek letter indicates the spinor index. Recalling that $G_{o}^{s} = (-k^{2} + m_{s}^{2})^{-1}$ one may construct from (5) a 4-vector potential, which has the structure $U_{s} - U_{v}$. Therefore in any perturbation expression, where the potential enters squared, one gets mixed coupling terms between the vector and scalar mesons. This usually used decoupling overlooks the problem of *meson initial correlations*, which is assumed to be zero when the mesonic degrees are eliminated by inverting their differential equation into an integral one, which was done using the free Green's function. Otherwise some initial terms have to occur.

We will proceed in an other way and develop a systematic many particle theory, which yields expressions without any mixed coupling terms with respect to the different meson contributions.

The physical properties of the system can be described by means of Green's functions or correlation functions. For nucleon system we define

$$
\begin{aligned}
iG &= \left\langle T\Psi\overline{\Psi}\right\rangle = iG_{++} & iG^{<} &= -\left\langle\overline{\Psi}\Psi\right\rangle = -iG_{+-} \\
i\overline{G} &= -\left\langle T_{-}\Psi\overline{\Psi}\right\rangle = iG_{--} & iG^{>} &= \left\langle\Psi\overline{\Psi}\right\rangle = iG_{-+}.
\end{aligned} \tag{6}
$$

and for the meson fields we introduce the correlation functions

$$id = \langle T\varphi\varphi'\rangle - \langle\varphi\rangle\langle\varphi'\rangle = id_{++}$$
$$i\overline{d} = -\langle T_{\varphi\varphi'}\rangle + \langle\varphi\rangle\langle\varphi'\rangle = id_{--}$$
$$id^< = -\langle\varphi'\varphi\rangle + \langle\varphi'\rangle\langle\varphi\rangle = -id_{+-}$$
$$id^> = \langle\varphi\varphi'\rangle - \langle\varphi\rangle\langle\varphi'\rangle = id_{-+}. \tag{7}$$

Here we used the convenient matrix - notation where the $\pm$ terms of the 2 x 2 correlation matrix will be signed by *latin* letters. For the reason of legibility we write down only the scalar meson equations in the following. The vector meson Green function carry spin indices. All final results will be presented for both, vector and scalar mesons.

Using the definition (6) and (7) the equations of motion for the different Green's functions can be derived. The equation of motion for the causal one reads e.g.

$$(i\gamma^\mu\,\partial_\mu - \kappa)\,G = \delta(1-1') + \frac{1}{i}\langle Tj_\varphi\psi\rangle \tag{8}$$

$$(\Box + m_s)\,d = -\delta\,(1-1') - \frac{1}{i}\langle Tj_\psi\varphi'\rangle \tag{9}$$

with j_φ and j_Ψ from eq. (2). Up to now we have not specified the meaning of the average. Solutions, which coincide to some special averages, have to be chosen by appropriate boundary conditions. For equilibrium, which corresponds to the average with the grand canonical equilibrium density operator, the famous KMS condition [28,19] holds

$$G^> \big|_{t=0} = \pm e^{\beta\mu} G^< \big|_{t'=-i\beta}. \tag{10}$$

In the nonequilibrium situation and for real time Green's functions this condition is not valid. Especially $G^<$ and $G^>$ are independent functions. One powerful possibility to derive real time Green functions is the condition of weakening of initial correlation [11], which is to be expressed in systems with finite densities. If the condition for the correlation time τ_{corr} and the mean free collision time τ_{coll}

$$\tau_{corr} \ll \tau_{coll}$$

is valid, we have a condition for the right sides of (8) and (9)

$$\lim_{t\to-\infty} \langle j_\varphi\psi\rangle = \langle\psi\overline{\psi}\rangle\left\{g_s\langle\Phi\rangle - g_v\gamma^\lambda\langle\varphi_\lambda\rangle\right\}. \tag{11}$$

This is an asymptotic condition, which breaks the time symmetry and provides irreversible evolution in nonequilibrium systems.

In the next step the correlated self energy Σ_c is introduced formally by subtracting the mean field parts 11) from the right side of eq. (8)

$$\frac{1}{i}\langle Tj_\varphi\psi\rangle - \lim_{t\to-\infty}\langle Tj_\varphi\psi\rangle \equiv \int_c \Sigma_c G. \tag{12}$$

The way of integration c has to be determined in such a way that (11) is fulfilled. This can be found by

$$\int_c d\overline{1}\Sigma(1,\overline{1})G(\overline{1},1') = \int_{-\infty}^{+\infty} d\overline{1}\left\{\Sigma(1,\overline{1})G(\overline{1},1') - \Sigma^<(1,\overline{1})G^>(\overline{1},1')\right\}$$

$$= \sum_{d=\pm}\int \Sigma_{+d}G_{d+}. \tag{13}$$

It is easy to see that the boundary condition is fulfilled, since the contribution (13) vanishes in the limit $t'_1 = t_1^\pm \to -\infty$. For the case $t_1 < t'_1$ (and vice versa) we can write e.g.

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$$\int\limits_{-\infty}^{+\infty} d\bar{1}\left\{\Sigma(1,\bar{1})G(\bar{1},1') - \Sigma^<(1,\bar{1})G^>(\bar{1},1')\right\} =$$

$$= \int_{-\infty}^{t_1} \Sigma^>(1,\bar{1})G^<(\bar{1},1') + \int_{t_1}^{t_1'} \Sigma^<(1,\bar{1})G^<(\bar{1},1') +$$

$$+ \int_{t_1'}^{\infty} \Sigma^<(1,\bar{1})G^>(\bar{1},1') - \int_{-\infty}^{\infty} \Sigma^<(1,\bar{1})G^>(\bar{1},1')$$

$$\to 0 \ for \ t_1 \to t_1 \to t_o = -\infty \tag{14}$$

If we split the last integral on the right of (13) into two parts according to

$$\int_{-\infty}^{+\infty} d\bar{t}_1 = \int_{-\infty}^{t_1'} d\bar{t}_1 + \int_{t_1'}^{+\infty} d\bar{t}_1$$

a contour of time integration follows which is equal to the Keldysh–contour [9,7].

Now we are able to enclose the equation of motion (8) resulting in the nonequilibrium Dyson equation as matrix equation (6) in the following form

$$(i\gamma^\mu\partial_\mu - \kappa)\,G_{bc}(11') = \delta_{bc}(11') + \int\limits_{-\infty}^{\infty} d\,\bar{\bar{1}}\,\Sigma_{b\bar{a}}\left(1\,\bar{\bar{1}}\right)G_{\bar{a}c}\left(\bar{\bar{1}}\,1'\right) \tag{15}$$

and for the meson correlation functions (9) one has

$$\left(\Box + m^2\right)d_{bc}(11') = \delta_{bc}(1 - 1') + \sum_d \int d2\Pi_{bd}(12)d_{dc}(21'). \tag{16}$$

Equations (eq11) and (16) are matrix equations, where latin letters remark $(+,-)$ in the following. To study the structure of the self energy Σ and the polarization function Π explicitly we want to use a technique developed by Schwinger [24]. Therefore we introduce an infinitesimal meson generating flux j_Φ for scalar mesons and j_λ for vector mesons, as a new type of interaction

$$L_{\text{int}} = -j_\Phi\Phi - j_\lambda\varphi^\lambda. \tag{17}$$

In our presented formalism it is not necessary to introduce a generating functional for nucleons. It turns out that the infinitesimal interaction (17) is sufficient to obtain the Kadanoff-Baym [28] equations. Therefore they are valid for any density or correlations in the system.

At this point it has to be remarked, that we suppress the nondiagonal terms of the vector mesons by choosing the generating functional in diagonal form. This is possible, because they must not contribute to physical observables such as S-matrices as a result of coupling to the baryon conserving flux. The influence to non – observable quantities such as self energy should be considered in principle. But they are neglected here for the reason of simplicity, which is in agreement with the treatment of Horowitz and Serot [29].

Now we introduce an interaction picture in respect to the infinitesimal interaction (17). Further, we distinguish between the upper and lower branch [18] of the contour because in such a way we can find relations for the time evolution operator of this (infinitesimal) interaction on the Keldysh contour, which reads

$$S_c = T_c\exp\left\{-ic\int\left(-j_\Phi\Phi - j_\mu\varphi^\mu\right)_c\right\}. \tag{18}$$

Thus special relations can be established by variational technique

$$\frac{\delta S_c}{\delta j_b} = -ib\,T_b\,\varphi'\,S_b\,\delta_{bc}, \tag{19}$$

where j stand for j_Φ or j_λ. Latin letters b,c remark $(+,-)$ respectivly. In a straight forward manner one can express all correlation functions (7) through variations with respect to j [18]. The causal Green's function e.g. can be expressed as

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$$i\frac{\delta <\varphi(1)>_+}{\delta j_+(1')} = \langle T_+\varphi(1)\varphi(1')\rangle - <\varphi(1)>_+<\varphi(1')>_+\equiv d_{++}. \tag{20}$$

With some manipulations the introduced polarization function (16) of mesons can be derived from (9) in the form

$$\Pi_{bd}(12) = -g^2\,b\sum_{a\bar{a}}\int Tr\left\{G_{ba}(1\bar{1})\Gamma_{a\bar{a}d}\left(\bar{1}\,\bar{\bar{1}}\,2\right)G_{\bar{a}b}\left(\bar{\bar{1}}\,1^+\right)\right\} \tag{21}$$

where we have introduced the vertex function

$$\Gamma_{a\bar{a}d}\left(\bar{1}\,\bar{\bar{1}}\,2\right) = -\frac{1}{g}\frac{\delta G_{a\bar{a}}^{-1}\left(\bar{1}\,\bar{\bar{1}}\right)}{\delta <\varphi(2)>_d}$$

$$= -i\delta_{a\bar{a}}\left(\bar{1}-\bar{\bar{1}}\right)\delta_{ad}\left(\bar{1}-2\right) + \frac{1}{g}\frac{\delta\sum_{a\bar{a}}\left(\bar{1}\,\bar{\bar{1}}\right)}{\delta<\varphi(2)>_d}. \tag{22}$$

The second line of (22) is quite easy to verify by means of (15). Concerning the nucleons we find that the right hand side of equation (8) could be expressed by the help of variations in such a way that the equation (8) takes the form

$$\left[i\gamma^\mu\delta_\mu - \kappa_0 + g_s <\phi>_b -g_v\langle\varphi_\lambda\rangle\gamma^\lambda\right]G_{bc}(11') = \delta_{bc}(11')$$

$$+\left\{\frac{bg_s}{i}\frac{\delta}{\delta j_{\phi b}(1)} - \frac{bg_v}{i}\gamma_\lambda\frac{\delta}{\delta j_{\lambda b}(1)}\right\}G_{bc}(11'). \tag{23}$$

After the introduction of the same vertex function (22), the structure of the self energy introduced in (15) can be written finally as

$$\Sigma_{b\bar{a}}(1\,\bar{\bar{1}}) = -\sum\int:\quad bg^2G_{ba}(1\bar{1})\Gamma_{a\bar{a}d}(\bar{1}\,\bar{\bar{1}}\,2)d_{db}(2\,1)\quad:. \tag{24}$$

Equations (15), (16), (21), (22) and (24) form a complete quantum statistical description of the many particle system in nonequilibrium and serves as a starting point for any further approximated treatment. We have to point out, that the different contributions of mesons are linearly additive in the self energies and therefore quadratic additive in the coupling constants as it is to be seen from (23). This differs from other treatments, where on the stage of eq. (2) the different parts are decoupled approximately yielding a linear additive coupling. Since we have not used any restriction or approximation in deriving the general structure of (23), it has to be considered as a more general treatment. The physical reason is, that during the derivation of the normal simple additive coupling initial correlations of different meson are neglected. By the scheme proposed here this is done carefully resulting in a quadratic additive behavior.

III. SPECTRAL INFORMATION

Because a complete description of a nonequilibrium situation demands *two* independent correlation functions instead of one, as it is the case in equilibrium, we have some relations between the *four* Green functions

$$G^r - G^> = G^a - G^< = \overline{G}$$
$$G^r + G^< = G^a + G^> = G, \tag{25}$$

where we introduce the retarded and advanced functions [9]

$$G^{r/a} = G_\delta \pm \Theta\left(\pm(x_0 - x_0')\right)\left\{G^> - G^<\right\}. \tag{26}$$

This function determines the spectral information [19] and we get the equation for them from (15)

$$(i\gamma^\mu\partial_\mu - \kappa)G^r = \delta(1\,1') + \int d\bar{1}\Sigma^r(1\bar{1})G^r(\bar{1}1'). \tag{27}$$

After the Wigner transformation, the variables are splitted in macroscopic and microscopic ones and it is assumed

$$\Sigma^r(X,p) \gg \left|\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial p_\mu}\Sigma^r(X,p)\right| \gg .... \tag{28}$$

This means that $\Delta x^\mu \Delta p_\mu \gg 1$ where the characteristic length Δp, at which the self energy varies in four–momentum space, corresponds to the inverse space–time interaction range. Therefore the approximation (29) demands the shortness of the space–time interaction range when compared with the systems space–time inhomogeneity scale [14].

All this is the physical background, when we applied the so called gradient expansion at this place, which yields now for (27)

$$(\gamma^\mu p_\mu - \kappa - \Sigma^r(pX))\, G^r(pX) = 1. \tag{29}$$

To make further progress, we write the self energy in characteristic parts in terms of the $\gamma-$matrices, which can be verified by general considerations [30,31]

$$\Sigma^r = \gamma^\mu \Sigma^r_\mu + I\Sigma^r_I. \tag{30}$$

Further it is useful to split up the self energy in real $Re\Sigma$ and imaginary Γ parts, between which exist the dispersion relation

$$Re\Sigma^r(pR\omega T) = P \int \frac{d\overline{\omega}}{2\pi} \frac{\Gamma(p\overline{\omega}RT)}{\omega - \overline{\omega}}. \tag{31}$$

Introducing the following medium dressed variables

$$\tilde{\mathbf{P}}_\mu = p_\mu - Re\Sigma_\mu$$
$$\tilde{\mathbf{M}} = \kappa + Re\Sigma_I$$
$$\tilde{\mathbf{G}} = \left(\gamma_0\, p_0\, \varepsilon + \frac{\Gamma}{2}\right)\left(\gamma\tilde{\mathbf{P}} + \tilde{\mathbf{M}}\right) \tag{32}$$

one obtains the complete spectral function from (29) as

$$A = i\left\{G^r(\omega + i\varepsilon) - G^a(\omega - i\varepsilon)\right\} = \left(\gamma^\mu \tilde{\mathbf{P}}_\mu + \mathbf{I}\tilde{\mathbf{M}}\right) \frac{2\tilde{\mathbf{G}}}{\left(\tilde{\mathbf{P}}^2 - \tilde{\mathbf{M}}^2\right)^2 + \tilde{\mathbf{G}}^2}. \tag{33}$$

This derivation underlines the correct expression using $\epsilon\gamma^0$ instead of ϵ in the denominator as it was pointed out by P. Henning [32]. In the case of vanishing damping Γ we get the spectral function, which determines the quasiparticle energies in the quasiparticle picture

$$A = 2\pi \left\{\gamma^\mu \tilde{\mathbf{P}}_\mu + I\, \tilde{\mathbf{M}}\right\} \delta\left(\tilde{\mathbf{P}}^2 - \tilde{\mathbf{M}}^2\right) \operatorname{sgn}\left(\tilde{\mathbf{P}}_0\right). \tag{34}$$

The δ Function in (34) represents the defining equation for the quasi–particle–energies

$$E_{1/2} = Re\Sigma^r_0 \pm \sqrt{(p_I - Re\Sigma^r_I)^2 + (\kappa_o + Re\Sigma_I)^2}\,\big|_{\omega = E_{1/2}} \tag{35}$$

This is a nonlinear equation by the many particle influence and shows immediately the particle/antiparticle symmetry. Therefore the single particle and antiparticle states become dressed by the medium and yield a natural generalization of the Dirac–Bruckner Theory. It can be seen that (35) is determined only by the retarded self energy, which yields a mass off-shell behavior of the quasiparticles. This is important to note for the discussion in chapter 4.

Let us now consider the meson equations, which read for the retarded Green's function written in operator notation, where integration about inner variables is assumed

$$\left(\Box + m_0^2\right)(-d^r) = 1 - \Pi^r d^r. \tag{36}$$

Moreover we have for the correlation function

$$\left(\Box + m_0^2\right)(-d)^{\gtrless} = -\Pi^r d^{\gtrless} - \Pi^{\gtrless} d^a. \tag{37}$$

Combining (36) and (37) we obtain the following important relation

$$-d^{\gtrless} = d^r\, \Pi^{\gtrless}\, d^a \tag{38}$$

which presents the generalized optical theorem [7,28]. Following the same arguments as deriving (29) one gets for the product $d^r d^a$ in gradient expansion

$$d^r d^a = \frac{1}{\left(\omega^2 - E_d^2\right)^2 + \left(\frac{\Gamma_\pi}{2} + 2\varepsilon\omega\right)^2}. \tag{39}$$

Here the quasi particle energy of mesons is introduced by

$$E_d^2 = p^2 + m^2 - Re\Pi. \tag{40}$$

The spectral function of mesons in the case of nearly vanishing damping reads

$$B = i\left(d^r - d^a\right) \longrightarrow 2\pi\delta\left(\omega^2 - E_d^2\right)\mathrm{sgn}(\omega) \qquad for \qquad \Gamma \to 0. \tag{41}$$

In static case with vanishing damping one obtains from (39) the Fourier–transformed Yukawa–potential for the meson exchange

$$d^r d^a \approx \frac{1}{g^4(4\pi)^4}V^2(p)$$

$$V(r) = g^2 \frac{e^{-r/r_0}}{r} \qquad r_0^{-2} = m^2 - Re\Pi. \tag{42}$$

When Π is determined by (21) we have the possibility to construct a self consistent system including fluctuation phenomena.

IV. KINETIC EQUATIONS

The starting point are the generalized KB equations [33] which are exact and read in the space–time notation

$$\left[ReG^{r^{-1}},\ G^{\lessgtr}\right] - \left[\Sigma^{\lessgtr},\ Re\,G\right] = \frac{1}{2}\left\{G^<\,\Sigma^>\right\} - \frac{1}{2}\left\{G^>,\,\Sigma^<\right\}. \tag{43}$$

Here we used operator notations, where the integration is assumed over inner variables and the brackets sign the commutator [] and anticommutator {} respectively.

Using the gradient expansion here we have a different physical content than the one yielding (29) and (36). If we suppose here

$$G(X,p) \gg \left|\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial p_\mu}G(X,p)\right| \gg ... \tag{44}$$

we have to demand that $\Delta X \Delta p \gg 1$. This would be justified in the case of one particle systems only by classical description. Because we have a many particle system G contains information averaged over space–time cells, which can be in principle smaller than the one determined by the single particle de Broglie wave length [12]. Because in nuclear physics time gradients occur, which are in principle nonvanishing, this assumption is the most restrictive one and is now under investigation and will soon be published.We have from (43) in $p\omega RT$ variables

$$\left[Re\,G^{r^{-1}},\ G^<\right] = G^<\ \Sigma^> - G^>\Sigma^< \tag{45}$$

where the bracket means the Poisson-bracket:

$$[A, B] = \frac{\partial}{\partial \omega}A\frac{\partial}{\partial T}B - \frac{\partial}{\partial \omega}B\frac{\partial}{\partial T}A + \bigtriangledown_R A \bigtriangledown_p B - \bigtriangledown_p A \bigtriangledown_R B. \tag{46}$$

As a first step of approximation we neglect higher vertex corrections in agreement with the gradient expansion and derive from (22) for the vertex function

$$\Gamma_{a\bar{a}d}\left(\bar{1}\ \bar{\bar{1}}\ 2\right) \approx \delta_{a\bar{a}}(1 - \bar{1})\delta_{ad}(\bar{1} - 2). \tag{47}$$

In the framework of this relativistic Random Phase Approximation (RRPA) we get from (15), (16), (21), (24), (38) and (47) the complete set of equations

$$\left[G^{r^{-1}}, G^< \right] (p\omega RT) = G^< \Sigma^> - G^> \Sigma^>$$

$$\Sigma^\lessgtr(xX) = -ig_0^2 \, G^\lessgtr \left(1 \; \bar{\bar{1}} \right) d^\lessgtr \left(\bar{\bar{1}} \; 1 \right)$$

$$d^\lessgtr(p\omega RT) = \frac{1}{|4\pi g_0^2|^2} V^2(p,\omega)\Pi^\lessgtr$$

$$\Pi^\lessgtr(xX) = ig^2 \, Tr \left\{ G^\lessgtr G^\lessgtr \right\}. \tag{48}$$

It has to be stressed that the dynamical potential introduced by the optical theorem (39) now contains an infinitesimal sum of nucleon fluctuations by the polarization function resulting in an effective meson exchange mass. Therefore this approximation can be considered as a modified first Born approximation including collective effects, especially density fluctuations.

The set of equations (48a-d) forms a closed set and determines the correlation function $G^\lessgtr$ and therefore the kinetic description proposed we find a connection between $G^\lessgtr$.

Therefore we proceed to the kinetic level of description by introducing the generalized distribution

$$G^>(p\omega RT) = -iA(p\omega RT)(1 - f(p\omega RT))$$
$$G^< (p\omega RT) = i \, A(p\omega RT) \, f(p\omega RT) \tag{49}$$

which is until now an exact variable change without approximations. Especially it fulfills the conditions (33).

As far as the generalized distribution should describe physical particles and antiparticles we use the Dirac interpretation in the quasiparticle picture. There an empty state of particle with negative energy equals to an antiparticle state with positive energy and we can write

$$f(-\omega) = 1 - F(\omega), \tag{50}$$

where F signs the antiparticle distribution. In equilibrium this is identical with the fact, that the chemical potential has to be chosen with opposite signs for particles and antiparticles, which follows immediately from the conserved baryon density [1].

With the help of (34) and (35) we obtain for $G^<$

$$G^< = \frac{i\pi}{\sqrt{\mathbf{P}_1^2 + \mathbf{M}^2}} \left\{ \left(\gamma^0 E_1 - \gamma^1 \mathbf{P}_1 + \mathbf{M}\right) \, f\left(E_1\right) \delta\left(\omega - E_1\right) \right.$$
$$\left. - \left(-\gamma^0 E_2 - \gamma^1 \mathbf{P}_1 + \mathbf{M}\right) f\left(-E_2\right) \delta\left(\omega + E_2\right) \right\}. \tag{51}$$

The choice of (50) coincides with Bezzerides and DeBois [18] if we set $F^<(\omega) = f(\omega)$ and $F^>(\bar\omega) = 1 - F(\omega)$.

If we now introduce (50) and (51) into the kinetic equation (48a) we have to perform the Poisson brackets on the left side carefully. After partial integration one can show, that the renormalization denominator, which arise from the spectral function (34) cancels exactly with the factors following from the gradient expansion on the left side so that the drift term takes the form

$$\frac{\partial}{\partial T} f + \nabla_R E \, \nabla_p \, f - \nabla_p E \, \nabla_R \, f.$$

we want to restrict to virtual meson exchange and neglect all density terms in the meson Green's function. After somewhat extensive but straight forward calculation using the set of equations (48a-d) we finally arrive at the kinetic equation for the particle distribution, if we integrate both sides over positive frequencies. The equation for antiparticles are obvious by replacing $f_p \longleftrightarrow F_p$.

We use for shortness the following abbreviation of the dynamically potential (39) without static approximation

$$V^2(p, \bar{p}, p_1, \bar{p}1) =$$
$$\left(2(p\bar{p}_1)(\bar{p}p_1) + 2(pp_1)(\bar{p}\bar{p}_1) - 2\kappa^2[(p_1\bar{p}_1) + (p\bar{p})] + 4\kappa^4 \right) V_v^2 (\bar{p} - p)$$
$$- (p\bar{p} + \kappa^2)(p_1\bar{p}_1 + \kappa^2)V_s^2 (\bar{p} - p) \tag{52}$$

As already mentioned following (24) no mixed terms between g_r, g_s occur in the cross sections due to the additive behaviour of the different self energies. The kinetic equation reads now

$$\frac{\partial}{\partial T}f + \nabla_R E \nabla_p f - \nabla_p E \nabla_R f = Tr \int \left[g^< \Sigma^> - g^> \Sigma^< \right] \frac{d\omega}{2\pi} +$$
$$+ \frac{\partial}{\partial T} Tr \int P \frac{1}{(\omega - \omega')^2} \left[g^< \Sigma^> - g^> \Sigma^< \right] \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} \tag{53}$$

with the collision integral up to first *two* order gradient expansions. The second part of the right side of (53) is obtained by the second order gradient expansion or equivalently by the expansion of the time retardation up to first order. It ensures the complete energy conservation [34], whereas the first part of (53) ensures only the quasiparticle energy. Because we restrict here to the case, where the quasiparticle picture is valid, only the first part will be discussed. In a later paper we will present the results obtained for the second part. The first part of the right side from (53) containes 8 processes. The first one, which describes elastic particle particle scattering reads

$$\int \frac{d\bar{p}^3}{(2\pi)^3} \frac{dp_1^3}{(2\pi)^3} \frac{d\bar{p}_1^3}{(2\pi)^3} (2\pi)^4 \delta^{(4)} \left(\bar{p}_1 + \bar{p} - p_1 - p \right) \frac{1}{\left| E_1 \bar{E}_1 E \bar{E} \right|} *$$
$$V^2(p, \bar{p}, p_1, \bar{p}_1) \left\{ f_p f_{p_1} N \left(f_{\bar{p}} f_{\bar{p}_1} \right) - f_{\bar{p}} f_{\bar{p}_1} N \left(f_p f_{p_1} \right) \right\}. \tag{54}$$

Here and in the following we used the Pauli blocking factors

$$N(F) = 1 - F$$
$$N(F\, f) = (1 - F)(1 - f)$$
$$N(F\, f\, G) = (1 - F)(1 - f)(1 - G).$$

The next two processes are the the crossing symmetric processes in the t and u channel. They are obvious by the crossing symmetric relations and describe the elastic particle- antiparticle scattering. The next 5 processes are only possible due to the off-shell character of the quasi-particle energies (35)

$$\int \frac{d\bar{p}^3}{(2\pi)^3} \frac{dp_1^3}{(2\pi)^3} \frac{d\bar{p}_1^3}{(2\pi)^3} (2\pi)^4 \frac{1}{\left| E_1 \bar{E}_1 E \bar{E} \right|} *$$
$$\left[- V^2(p, \bar{p}, -p1, \bar{p}1) \delta^{(4)} \left(\bar{p}_1 + p_1 + \bar{p} - p \right) \left\{ f_p N \left(f_{\bar{p}} F_{p_1} f_{\bar{p}_1} \right) - f_{\bar{p}} f_{\bar{p}_1} F_{p_1} \right\} \right.$$
$$- V^2(p, \bar{p}, p_1, -\bar{p}_1) \delta^{(4)} \left(-\bar{p}_1 - p_1 + \bar{p} - p \right) \left\{ f_p f_{p_1} F_{\bar{p}_1} - N \left(f_p f_{p_1} F_{\bar{p}_1} \right) f_{\bar{p}} \right\}$$
$$- V^2(p, \bar{p}_1, p_1, -\bar{p}) \delta^{(4)} \left(-\bar{p} - p_1 + \bar{p}_1 - p \right) \left\{ f_p f_{p_1} F_{\bar{p}} - N \left(f_p f_{\bar{p}_1} F_{\bar{p}} \right) f_{\bar{p}} \right\}$$
$$- V^2(p, -\bar{p}, -p_1, -\bar{p}_1) \delta^{(4)} \left(-\bar{p}_1 + p_1 - \bar{p} - p \right) \left\{ f_p F_{\bar{p}} F_{\bar{p}_1} - F_{P_1} N \left(f p F_{\bar{p}} F_{\bar{p}_1} \right) \right\}$$
$$+ V^2(p, -\bar{p}, p_1, -\bar{p}_1) \delta^{(4)} \left(-\bar{p}_1 - p_1 - \bar{p} - p \right) *$$
$$\left. \left\{ f_p F_{\bar{p}} f_{p_1} F_{\bar{p}_1} - (1 - f_p)(1 - F_{\bar{p}})(1 - f_{p_1})(1 - F_{\bar{p}_1}) \right\} \right]. \tag{55}$$

Analyzing the energy momentum conserving δ-function on finds in connection with (35) that these processes occur if the c.m. energy $\sqrt{s}$ fulfills the condition

$$\sqrt{s} = -2Re\Sigma_o^r. \tag{56}$$

This means, that in a nucleus system, where the zero part of the selfenergy become negative, new reaction channels will be opened by many particle influence. One may think on pseudoscalar and pseudovector meson coupling, where the absence of mean field terms of the vector mesons ensures the condition (56).

Further, it can be seen explicitly that the relativistic treatment yields some forefactors (52) to the dynamical potentials (39). The dynamical potentials have to be determined selfconsistently with the kinetic equation by means of (53) and the polarization function (48d).

Therefore we arrived at a generalized relativistic Lenard-Balescu equation. The dynamical potentials reflect the influence of virtual mesons and are written for each process with the influence of the dynamical density fluctuations by (39).

The first 3 terms are presenting the elastic scattering between particles and antiparticles respectively. The next 5 processes are only possible, if the particles are off shell, which is ensured by the quasiparticle energies shown in (35).

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This can be understood as particle creation and destruction by means of virtual meson exchange. It is important to recognize, that these processes are special effects of many particle treatment and are forbidden by momentum and energy conserving δ-functions in the case of vanishing many particle influence.

The equation (53) possesses all properties of a selfconsistent quantum–mechanical kinetic equation which describes the relativistic behaviour of a meson-nucleon system from the many particle point of view. Particularly, it contains Pauli blocking and inelastic processes by the medium. This is the main result of the present paper and should be the starting point to kinetic description of hot nuclear matter.

To complete we give the in medium cross sections obtained for the elastic process (54). For shortness we denote $Re\Sigma_o^r = \Delta$ and $\tilde{\kappa} = \kappa + Re\Sigma_i^r$. The proton-proton cross section reads

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} = \frac{1}{32\pi^2} \Big\{ & \\
& \frac{2g_v^4}{s(m_v^2+t)^2}\Big[(\Delta(\sqrt{s}+\Delta)+\tilde{\kappa}^2-\frac{u}{2})^2 + (\Delta(\sqrt{s}+\Delta)-\tilde{\kappa}^2+\frac{s}{2})^2 \\
& -2\tilde{\kappa}^2(\Delta(\sqrt{s}+\Delta)-\frac{t}{2})\Big] \\
& - \frac{2g_s^4}{s(m_s^2+t)^2}(\Delta(\sqrt{s}+\Delta)+2\tilde{\kappa}^2-\frac{t}{2})^2 \Big\} .
\end{aligned}
\tag{57}
$$

Here s, t, u are the Mandelstam variables. This means that the in medium cross sections are effected by the dressed Baryon mass and an additional term proportional to the vector part of the self energy. If we had used the static approximation for the potential 42 we would have obtained $-t$ instead of t in the denominator.

The in-medium proton-antiproton elastic cross sections as well as the discussed medium caused inelastic ones can be derived from the corresponding collision integrals (54) and (55) in the same way. It turns out that they are identical to the cross sections one obtains from (57) by using crossing symmetric relations in the t or u channel or to inelastic crossing. This means that we end up with an expression for the medium dependent cross section for nucleons, which possess crossing symmetries and includes an infinitesimal series of meson interactions by the RPA polarization function resulting in an effective meson mass (40),

V. CONCLUSION

We have analysed the structure of equations for a nonequilibrium situation consisting in a relativistic nucleon-meson system. New features arise due to the consequent nonequilibrium treatment as pair creation and destruction and dynamically interacting meson masses. With one formalism of real time Green function one can determine the self energy, the quasiparticle behaviour and kinetic equations of the system and therefore the scattering measures. The kinetic equations are derived of Lenard Balescu type by V_s approximation which coincide with the RPA screened potentials in nonrelativistic treatment. These obtained potentials may serve as a starting point for constructing optical potentials [14]. On the other hand in-medium cross sections can be derived from the kinetic equation, which show no mixed terms of mesons contributions. This follows immediately from the more general decoupling of equations by the self energies. In a forthcoming work we will present the numerical results for the in-medium cross sections derived from (53) for the now opened inelastic scattering channel.

VI. ACKNOWLEDGEMENT

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